Please note that a typographical error occurred in equation 3 in this paper. Here is the correct equation for calculating efficiency, and its derivation.

Start with the equation relating efficiency to the cycle number at each threshold, A and B (cycle $C_{t,A}$ and $C_{t,B}$):

$$R_A = R_0 \cdot (1+E)^{C_{t,A}} \quad \text{......(1)}$$

and

$$R_B = R_0 \cdot (1+E)^{C_{t,B}} \quad \text{......(2)}$$

Express this as a ratio of R_A / R_B:

$$\frac{R_A}{R_B} = \frac{(1+E)^{C_{t,A}}}{(1+E)^{C_{t,B}}} \quad \text{......(3)}$$

(The R_0 terms just cancel out). Both sides of this equation are <1.

This can be re-written as:

$$\frac{R_A}{R_B} = (1+E)^{C_{t,A}-C_{t,B}} \quad \text{......(4)}$$

Note that here that we assume that *E* is the same in equation (1) and (2); essentially we are calculating the efficiency between cycle $C_{t,A}$ and $C_{t,B}$. If cycle $C_{t,A}$ and $C_{t,B}$ are fairly close together we can assume that the efficiency is not changed during this period. For bigger gaps between cycle $C_{t,A}$ and $C_{t,B}$ this is likely to be untrue, and the efficiency calculated will be a some sort of average efficiency over the period).

so:

$$\left(\frac{R_{n,A}}{R_{n,B}}\right)^{\frac{1}{C_{t,A}-C_{t,B}}} = (1+E) \quad \text{......(5)}$$

or

$$E = \left(\frac{R_{n,A}}{R_{n,B}}\right)^{\frac{1}{C_{t,A}-C_{t,B}}} - 1 \quad \text{......(6)}$$

which is the correct form of the equation.
In Liu and Saint (2002) Anal Biochem. 2002 Mar 1;302(1):52-59, the correct equation was used in the simulation and calculations; a typo crept into the manuscript in equation 3, and unfortunately went undetected.